

Test 2 Numerical Mathematics 2

April 13, 2023

Duration: 1 hour.

In front of the questions one finds the points. The sum of the points plus 1 gives the end mark for this test.

1. Consider the 100×100 matrix

$$A = \begin{bmatrix} -1 & 101 & & & & & \\ -99 & -2 & 101 & & & & \\ & -99 & -2 & 101 & & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & -99 & -2 & 101 \\ & & & & & -99 & -2 \end{bmatrix}.$$

- (a) [1] Show that A is irreducible.
- (b) [2] Write A as the sum of a symmetric and a skew-symmetric matrix. Consider the symmetric part and localize its eigenvalues by the Gershgorin theorems. And similar for the skew-symmetric part of A
- (c) [1] According to Bendixson's theorem where are the eigenvalues of A located in the complex plane based on the results in the previous part?
2. [2] Let A be a real symmetric matrix. Let x , with $\|x\|_2 = 1$, and θ be a Ritz pair obtained from the Lanczos method. Show that

$$\|Ax - \theta x\|_2 > \min_{\lambda \in \sigma(A)} |\lambda - \theta|.$$

3. Consider the three matrices below

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 4.6792 & .2979 & 0 \\ .2979 & 3.0524 & .0274 \\ 0 & .0274 & 1.2684 \end{bmatrix}, \quad \begin{bmatrix} 4.7104 & .1924 & 0 \\ .1924 & 3.0216 & -.0115 \\ 0 & -.0115 & 1.2680 \end{bmatrix}$$

which are respectively the original matrix and two subsequent iterates in the QR-method. Moreover, it is given that the eigenvalues of the original matrix are 4.7321, 3.0 and 1.2679.

- (a) [1] How is the QR-method defined? Where does it, for general real matrices, converge to?
- (b) [1] Explain the reduction factor of the off-diagonal elements from the middle to the right matrix.
- (c) [1] Suppose we apply a QR-step including shift to the middle matrix. By which factor will the (3,2) element decrease approximately?